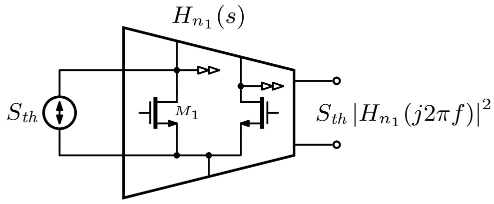
Each MOSFET contributes with a current PSD due to thermal noise:

$$S_{th} = 4kT\frac{\gamma}{\alpha}g_{mk} \quad \left[\frac{\mathrm{A}^2}{\mathrm{Hz}}\right].$$

Take e. g. the differential pair M_1, M_2; the transfer function between noise current and output voltage will be similar to

$$H_{n1}(s) \triangleq \frac{v_{no}}{i_n} = \frac{R_O}{1+\frac{s}{\omega_d}} \quad [\Omega]$$

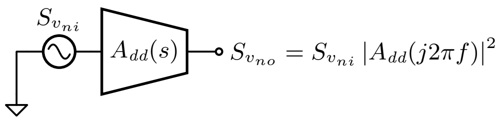
where $\omega_d = 2\pi f_d \approx \frac{1}{R_O C_L'}$ is the dominant pole of the open-loop OTA.



Therefore, the total voltage PSD seen at the output of the open-loop OTA is

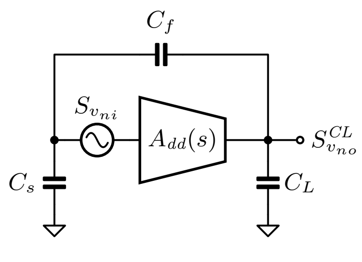
$$S_{v_{no}} = \sum_k S_{th}\,|H_{nk}(j2\pi f)|^2 = 8kT\frac{\gamma}{\alpha}g_{m1}\frac{R_O^2}{1+\left(\frac{f}{f_d}\right)^2} \quad \left[\frac{\mathrm{A}^2}{\mathrm{Hz}}\cdot\Omega^2 = \frac{\mathrm{V}^2}{\mathrm{Hz}}\right]$$

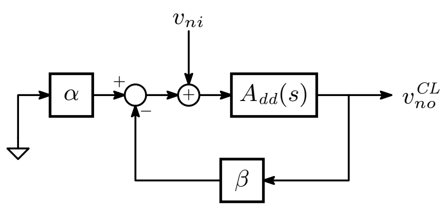
which is associated to an equivalent voltage PSD $S_{v_{ni}}$ applied to the input of the OTA, whose open-loop transfer function is $A_{dd}(s) \approx \frac{g_{m1}R_O}{1+s/\omega_d}$ under the dominant pole assumption:



$$\Longrightarrow \quad S_{v_{ni}} = \frac{S_{v_{no}}}{|A_{dd}(j2\pi f)|^2} = 8kT\frac{\gamma}{\alpha}\cdot\frac{1}{g_{m1}} \quad \left[\frac{\mathrm{V}^2}{\mathrm{Hz}}\right],$$

therefore, applying feedback theory,





$$S_{v_{no}}^{CL} = S_{v_{ni}}\left|\frac{1}{\beta}\cdot\underbrace{\frac{T_0}{1+T_0}}_{\sim 1}\cdot\frac{1}{1+j\frac{f}{f_c}}\right|^2 \quad \left[\frac{\mathrm{V}^2}{\mathrm{Hz}}\right],$$

where the pole of the feedback configuration is $f_c = (1+T_0)f_d \approx \beta g_{m1}R_O f_d = \beta\frac{g_{m1}}{2\pi C_L'}$.

Finally, the noise variance is

$$\sigma_{v_{no}}^2 = \int_0^{+\infty} S_{v_{no}}^{CL}(f)\,\mathrm{d}f = 8kT\frac{\gamma}{\alpha}\cdot\frac{1}{g_{m1}}\cdot\frac{1}{\beta^2}\cdot f_c\underbrace{\int_0^{\infty}\frac{\mathrm{d}f}{1+f^2}}_{\pi/2} = \frac{2kT}{\beta}\frac{\gamma}{\alpha}\cdot\frac{1}{C_L'} \quad [\mathrm{V}^2].$$

Updated September 11, 2022