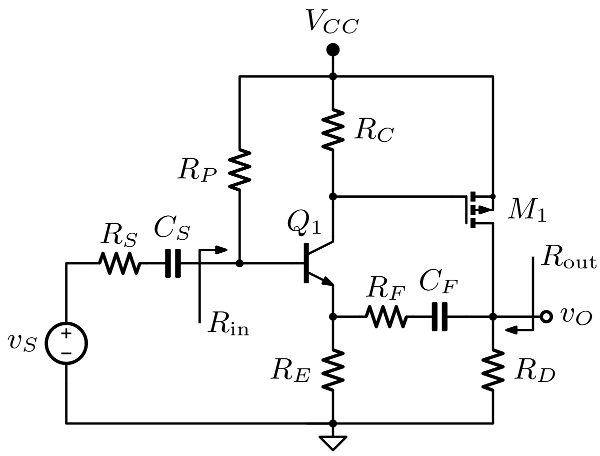
Exercise 1. Parameters at 25 °C:

- $V_{CC} = 12\,\mathrm{V}$;
- $R_S = 1\,\mathrm{k\Omega}$, $R_P = 1.2\,\mathrm{M\Omega}$, $R_C = 10\,\mathrm{k\Omega}$, $R_E = 220\,\Omega$, $R_F = 1.2\,\mathrm{k\Omega}$, $R_D = 2.2\,\mathrm{k\Omega}$;
- $C_S = 1\,\mathrm{\mu F}$, $C_F = 220\,\mathrm{\mu F}$;
- Q_1 (npn BJT): $V_{BE} = 0.7\,\mathrm{V}$, $\beta_F = \beta_0 = 100$, $r_o = +\infty$;
- M_1 (e-PMOS): $V_t = -3\,\mathrm{V}$, $I_{DSS} = 0.3\,\mathrm{mA}$, $r_o = +\infty$.



1. *Bias point of the devices.*

From the leftmost branch of the circuit (the one polarizing Q_1), the KVL gives Q_1's base current I_B:

$$V_{CC} = R_P I_B + V_{BE} + R_E(\beta_F + 1) I_B \implies$$
$$\implies I_B = \frac{V_{CC} - V_{BE}}{R_P + R_E(\beta_F + 1)} = \frac{12\,\mathrm{V} - 0.7\,\mathrm{V}}{1.2\,\mathrm{M\Omega} + 220\,\Omega \cdot 101} \simeq 9.245\,\mathrm{\mu A},$$

therefore the collector current is $I_C = \beta_F I_B \simeq 924.5\,\mathrm{\mu A}$.

Since the gate of M_1 drains no current, Q_1's collector current I_C flows also on the resistor R_C and the KVL on this branch allows to calculate V_{CE}:

$$V_{CC} = R_C \beta_F I_B + V_{CE} + R_E(\beta_F + 1) I_B \implies$$
$$\implies V_{CE} = V_{CC} - [R_C\beta_F + R_E(\beta_F + 1)] I_B =$$
$$= 12\,\mathrm{V} - [10\,\mathrm{k\Omega} \cdot 100 + 220\,\Omega \cdot 101] \cdot 9.245\,\mathrm{\mu A} =$$
$$\simeq 2.55\,\mathrm{V}.$$

Now, V_{GS} is the voltage across R_C, i. e. $V_{GS} = -R_C I_C \approx -9.245\,\mathrm{V}$, therefore

$$I_{DS} = I_{DSS}\left(1 - \frac{V_{GS}}{V_t}\right)^2 = 0.3\,\mathrm{mA}\left(1 - \frac{-9.245\,\mathrm{V}}{-3\,\mathrm{V}}\right)^2 \simeq 1.300\,\mathrm{mA}$$
$$V_{DS} = -(V_{CC} - R_D I_{DS}) = -12\,\mathrm{V} + 2.2\,\mathrm{k\Omega} \cdot 1.300\,\mathrm{mA} \simeq -9.14\,\mathrm{V}.$$

Therefore, the operating points are $Q_1(924.5\,\mathrm{\mu A}, 2.55\,\mathrm{V})$ and $M_1(1.300\,\mathrm{mA}, -9.14\,\mathrm{V})$.

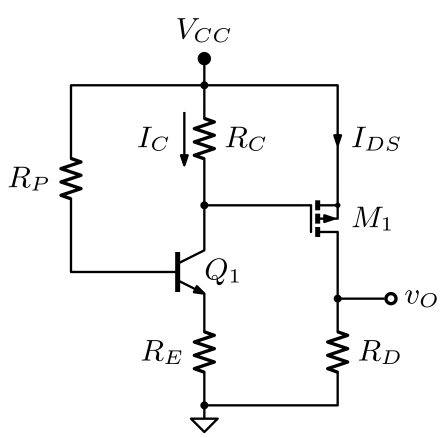


Figure 1: Bias point analysis of the circuit.

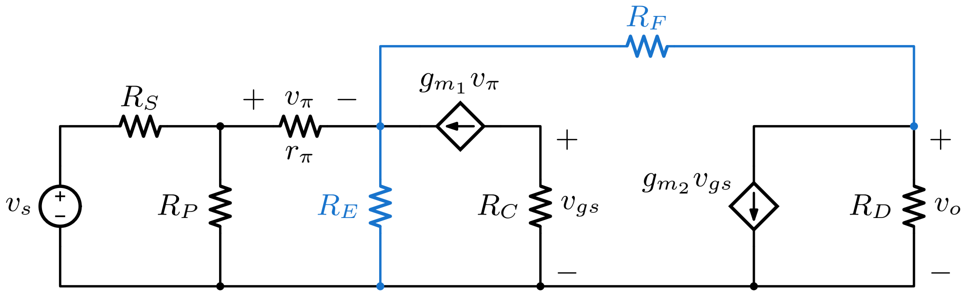


Figure 2: Small signal model of the circuit, where the feedback network is highlighted in blue.

2. *Equivalent small signal circuit, according to feedback theory.*

   From the operating points of the two transistor, the small signal parameters are:

$$g_{m_1} = \frac{I_C}{V_T} = \frac{924.5\,\mu\mathrm{A}}{26\,\mathrm{mV}} \simeq 35.6\,\mathrm{mS}, \qquad r_\pi = \frac{\beta_0}{g_{m_1}} = \frac{100}{35.6\,\mathrm{mS}} \simeq 2812\,\Omega$$

$$g_{m_2} = \frac{2}{|V_t|}\sqrt{I_{DSS} I_{DS}} = \frac{2}{3\,\mathrm{V}}\sqrt{0.3\,\mathrm{mA} \cdot 1.300\,\mathrm{mA}} \simeq 416\,\mu\mathrm{S}.$$

   Since the amplifier uses voltage sampling and voltage mixing, the equivalent β-network model is pictured in Figure 3, where

$$\begin{cases} v_1 = \beta v_2 + R_{11} i_1 \\ i_2 = \dfrac{v_2}{R_{22}} + \delta i_1 \end{cases} \implies \begin{aligned} \beta &\triangleq \left.\frac{v_1}{v_2}\right|_{i_1=0} = \frac{R_E}{R_F + R_E} \simeq 0.155 \\ R_{11} &\triangleq \left.\frac{v_1}{i_1}\right|_{v_2=0} = R_E \parallel R_F \simeq 186\,\Omega \\ R_{22} &\triangleq \left.\frac{v_2}{i_2}\right|_{i_1=0} = R_F + R_E = 1420\,\Omega \\ \delta &\triangleq \left.\frac{i_2}{i_1}\right|_{v_2=0} = -\frac{R_E}{R_F + R_E} \simeq -0.155. \end{aligned}$$

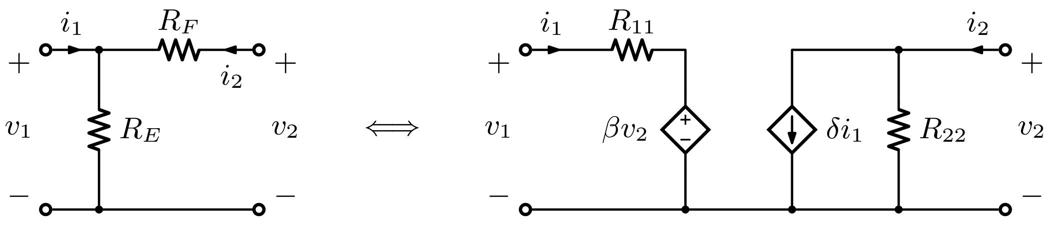


Figure 3: Equivalent β-network model for voltage sampling and voltage mixing.

   As usual, the effect of the generator δi_1 is negligible, therefore the β-network can be considered unidirectional. In Figure 4 the β-network is shown separated from the open-loop amplifier.

3. *Closed loop gain* $A_F \triangleq \frac{v_o}{v_s}$.

   Since the circuit is a voltage amplifier according to feedback theory, the closed-loop voltage gain A_F can be calculated from the open-loop gain A_{OL} as following:

$$A_F = \frac{A_{OL}}{1 + \beta A_{OL}}.$$

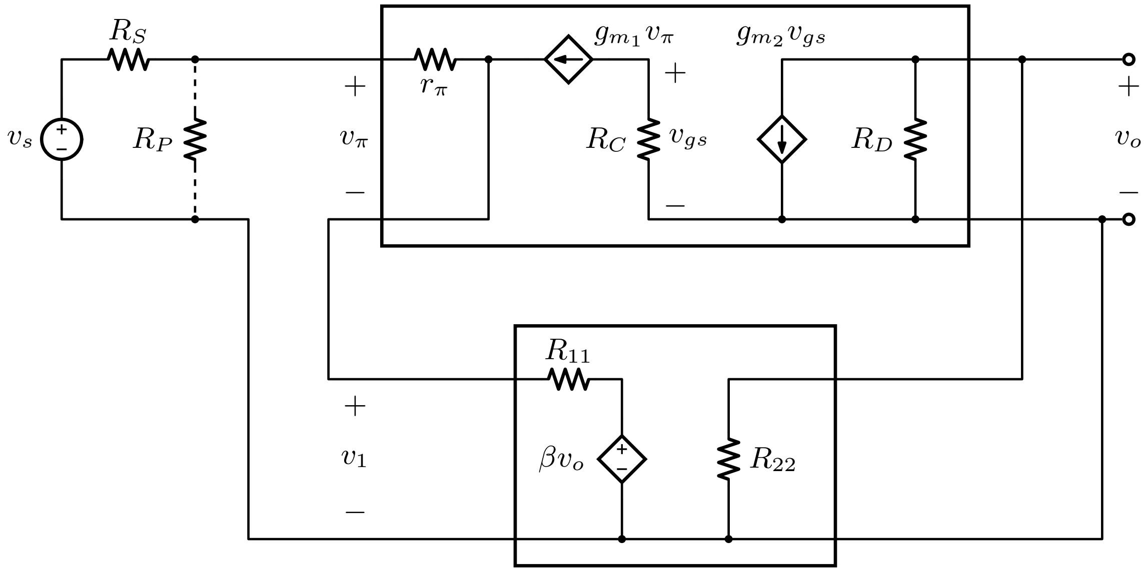


Figure 4: Small signal model according to feedback theory, with the equivalent β-network model separated from the open-loop amplifier and $\delta \approx 0$. R_P is negligible, because $R_P \gg R_S$.

Analysing the two stages of the amplifier, we find the open-loop voltage gain A_{OL}:

$$A_{OL} \triangleq \left.\frac{v_o}{v_s}\right|_{\beta \equiv 0} = \frac{\cancel{R_{\text{in}}^{CE}}}{R_s + R_{\text{in}}^{CE}} \cdot \overbrace{\left[-g_{m1}R_C\frac{r_\pi}{\cancel{r_\pi + R_{11}(\beta_0+1)}}\right]}^{A_v^{CE}} \cdot \overbrace{\left[-g_{m2}(R_D \| R_{22})\right]}^{A_v^{CS}} =$$

$$= \frac{r_\pi \cdot g_{m1}R_C \cdot g_{m2}(R_D \| R_{22})}{R_s + r_\pi + R_{11}(\beta_0+1)} = \frac{100 \cdot 10\,\text{k}\Omega \cdot 416\,\mu\text{S} \cdot (2.2\,\text{k}\Omega \parallel 1.42\,\text{k}\Omega)}{1\,\text{k}\Omega + 2812\,\Omega + 186\,\Omega \cdot 101} \simeq 15.89$$

Therefore,

$$A_F = \frac{A_{OL}}{1 + \beta A_{OL}} = \frac{15.89}{1 + 0.155 \cdot 15.89} \simeq 4.59.$$

4. *Input resistance* R_{in}.

   The open-loop input resistance is

$$R_{\text{in}}^{OL} = R_S + R_{\text{in}}^{CE} = R_S + r_\pi + R_{11}(\beta_0+1) \simeq 22.6\,\text{k}\Omega,$$

   where R_{in}^{CE} is the input resistance of the common emitter stage; therefore,

$$R_{\text{in}} = R_{\text{in}}^{OL}(1 + \beta A_{OL}) - R_S = 22.6\,\text{k}\Omega \cdot (1 + 0.155 \cdot 15.89) - 1\,\text{k}\Omega \simeq 77.3\,\text{k}\Omega.$$

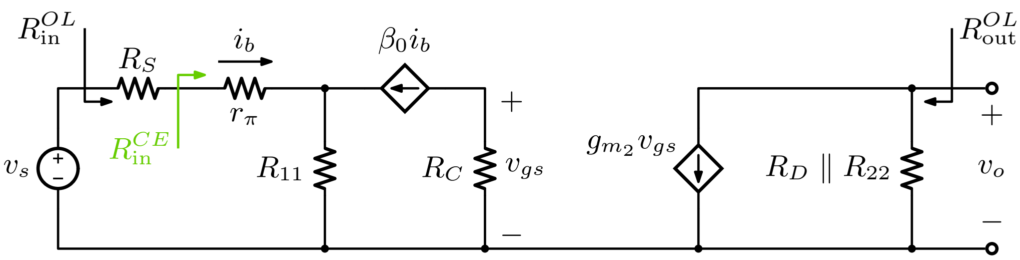
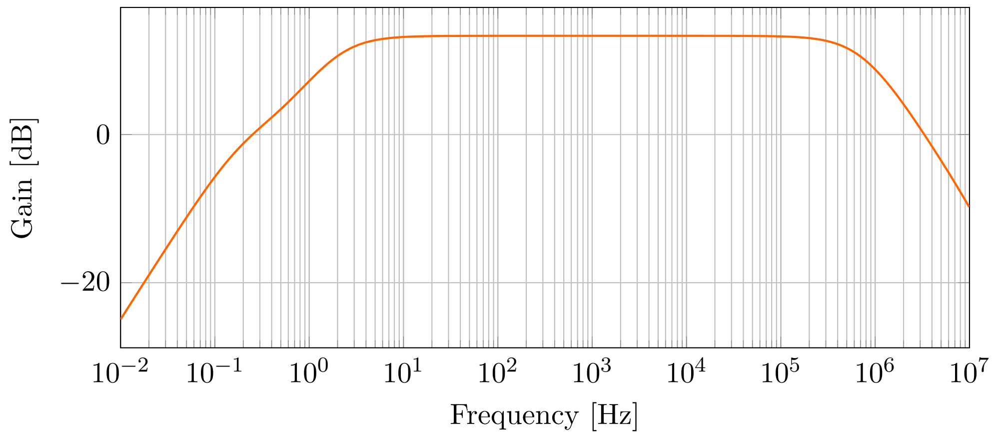


Figure 5: Open-loop small signal model of the amplifier, including loading effects of the β-network.

Figure 6: Simulated AC gain.



5. *Output resistance* R_{out}.

   From inspection, the open-loop output resistance is $R_{\text{out}}^{OL} = R_D \parallel R_{22} = 863\,\Omega$; therefore,

$$R_{\text{out}} = R_{\text{out}}^{F} = \frac{R_{\text{out}}^{OL}}{1+\beta A_{OL}} = \frac{863\,\Omega}{1+0.155\cdot 15.89} \simeq 249\,\Omega.$$

6. *Estimation of the lower cut-off frequency* f_L*, under the dominant pole assumption at low frequency.*

   Using the SCTC method,

$$\omega_L \approx \frac{1}{R_S^{SC} C_S} + \frac{1}{R_F^{SC} C_F}$$

   where R_S^{SC} is the resistance seen from C_S when C_F is short-circuited, and R_F^{SC} is the resistance seen from C_F when C_S is shorted:

$$R_S^{SC} = R_{\text{in}}^{OL} \simeq 22.6\,\text{k}\Omega$$

$$R_F^{SC} = R_F + R_{\text{out}}^{OL} + R_E \| R_{\text{out}}^{CC} \simeq 2.1\,\text{k}\Omega, \qquad \text{where } R_{\text{out}}^{CC} = \frac{r_\pi + R_S}{\beta_0 + 1}.$$

   Therefore,

$$f_L^{OL} \approx \frac{1}{2\pi}\left(\frac{1}{R_S^{SC} C_S} + \frac{1}{R_F^{SC} C_F}\right) \simeq 7.04\,\text{Hz} + 0.34\,\text{Hz} = 7.38\,\text{Hz}$$

   and the closed-loop lower frequency is

$$f_L^{CL} = \frac{f_L^{OL}}{1+\beta A_{OL}} = \frac{7.38\,\text{Hz}}{1+0.155\cdot 15.89} \simeq 2.13\,\text{Hz},$$

   while simulation provides 1.85 Hz, because we assumed a dominant low-frequency pole, but from the Bode diagram in Figure 6 this is not completely verified.

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